Bird and Runner Problem

Part A

runner distance: x

bird total distance: $L + (L - x)$

$$d_b = L + (L - x)$$

$$d_r = x$$

$$d_b = 2d_r$$

(bird is 2x faster)

$$d_b = 2d_r$$

$$L + (L - x) = 2x$$

$$2L - x = 2x$$

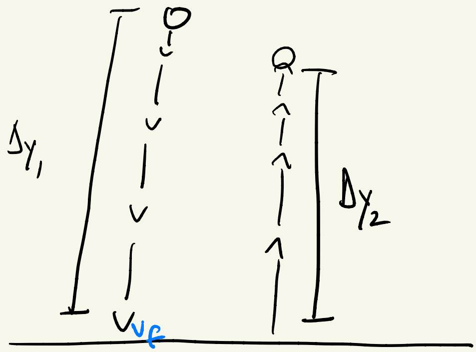
$$2L = 3x \Rightarrow x = \frac{2L}{3}$$

Part B

Bird is 2x faster,

$$d_b = 2d_r$$

Dropped Tennis Ball Problem



Part A

Δy_1

$v_0 = 0$

a

$v_f = ?$

1) Find Δt

2) use $v_f = v_0 + a\Delta t$

(negative)

Free fall:

$$\Delta y = \frac{1}{2} a \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Part B

$v_0 = ?$

Δy_2

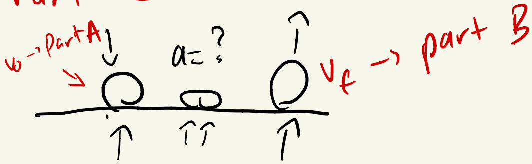
$v_f = 0$

a

$$v_f^2 = v_0^2 + 2a\Delta y$$

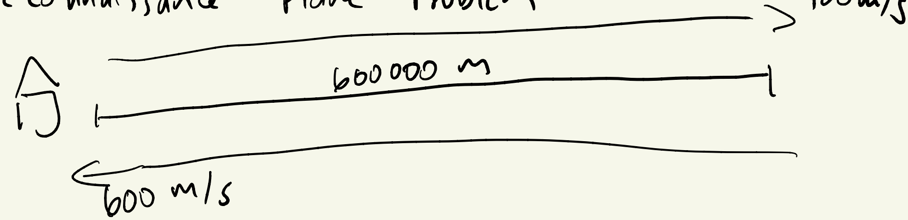
$$v_0 = \sqrt{v_f^2 - 2a\Delta y}$$ (positive)

Part C



$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{\Delta t}$$

Reconnaissance Plane Problem



1) Find Δt for both trips

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v}$$

Forward trip: $\Delta t_f = \frac{600000}{400}$

Return trip: $\Delta t_r = \frac{600000}{600}$

2) Find $\bar{V}_{avg}$ $\quad v = \frac{\Delta x}{\Delta t} = \frac{2(600000)}{(\Delta t_f + \Delta t_r)} = \bar{V}$

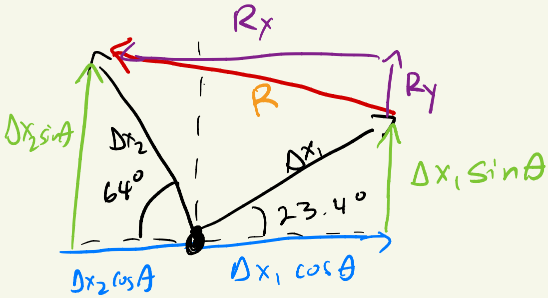
$$400 < \bar{V} < 600$$

Two Planes Problem

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v\,\Delta t$$

Δx_1

Δx_2



$$\theta = 180 - 116 = 64°$$

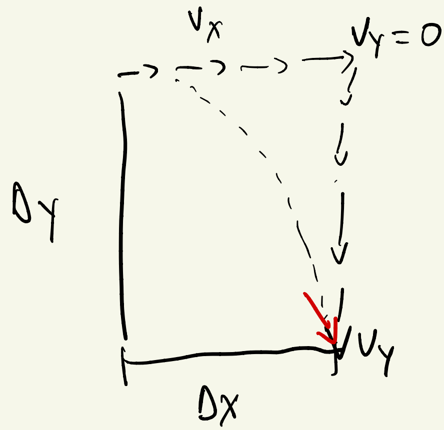
$$R_y = \Delta x_2 \sin\theta - \Delta x_1 \sin\theta$$

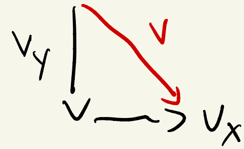
$$R_x = \Delta x_1 \cos\theta + \Delta x_2 \cos\theta$$

$$R = \sqrt{R_x^2 + R_y^2}$$

Projectile Final Velocity Problem

$$v = \sqrt{v_y^2 + v_x^2}$$





X

1) Find Δt

$$\Delta t = \sqrt{\frac{2\Delta y}{g}}$$

$$v_x = \frac{\Delta x}{\Delta t}$$

Y

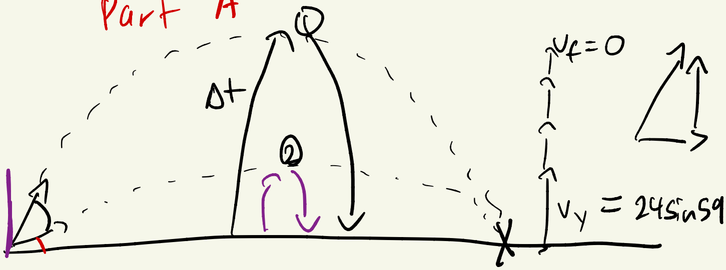
$v_0 = 0$ $\quad v_f = v_y$

$a = g$

$$v_f = v_0 + a\Delta t$$

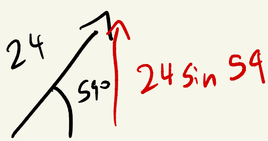
Two Snowballs Problem

Part A



Part B

t_1



$V_0 = V_y$

$V_f = 0$

$a = g$

$$V_f = V_0 + a\Delta t$$

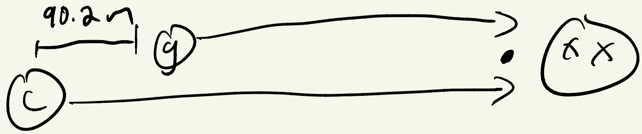
total $\Delta t = 2\Delta t = \Delta t_{total_1}$

$\Delta t_{total_2} = ?$

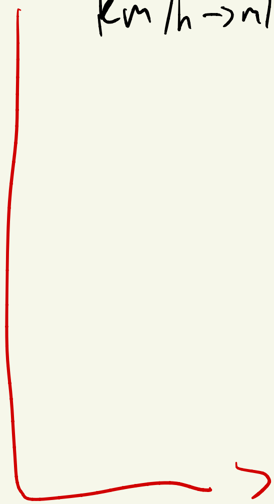
Subtract for answer

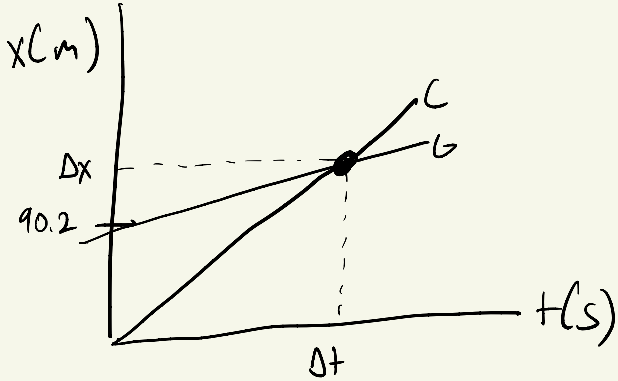
Cheetah & Gazelle Problem

Part A



km/h → m/s





$y = mx + b$

cheetah → $\Delta x = v_C \Delta t$

gazelle → $\Delta x = v_G \Delta t + 90.2$

$v_C \Delta t = v_G \Delta t + 90.2$

$\Delta t (v_C - v_G) = 90.2$

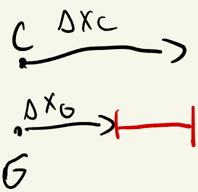
Part B

Δx in 7.5 s

C: $v_C \Delta t = \Delta x_C$

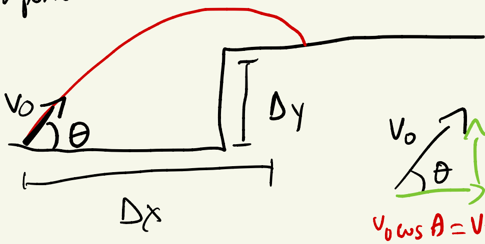
G: $v_G \Delta t = \Delta x_G$

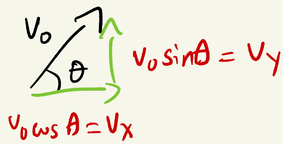
$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = v \Delta t$



Salmon Problem

1) Break v_0 into x and y components





2) Find Δt

$$V_x = \frac{\Delta x}{\Delta t}$$

$$\Rightarrow \Delta t = \frac{\Delta x}{V_x} = \frac{\Delta x}{V_0 \cos\theta}$$

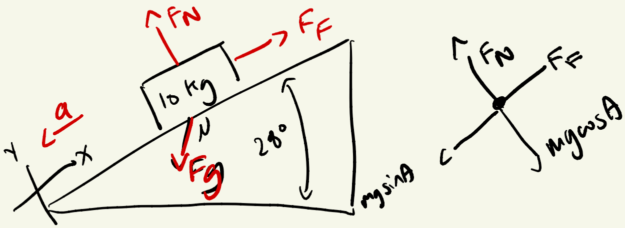
3) $\Delta y = V_0 \Delta t - \frac{1}{2} g \Delta t^2$

$$\Delta y = V_0 \sin\theta \left(\frac{\Delta x}{V_0 \cos\theta}\right) - \frac{g\left(\frac{\Delta x}{V_0 \cos\theta}\right)^2}{2}$$

$$= \frac{V_0 \sin\theta \, \Delta x}{V_0 \cos\theta} - \frac{g \Delta x^2}{2 v_0^2 \cos^2\theta} = \tan\theta \, \Delta x - \frac{g \Delta x^2}{2 v_0^2 \cos^2\theta}$$

$$\Rightarrow V_0^2 = \frac{-g \Delta x^2}{2\cos^2\theta\,(\Delta y - \tan\theta \, \Delta x)}$$

Block at Rest on Plane Problem



Part A

$$\Sigma F = 0 = F_F - mg\sin\theta \Rightarrow F_F = mg\sin\theta$$

Part B

$$F_F = \mu F_N \Rightarrow (mg\sin\theta) = \mu(mg\cos\theta)$$

$$\mu = \tan\theta \Rightarrow \theta = \tan^{-1}(\mu)$$

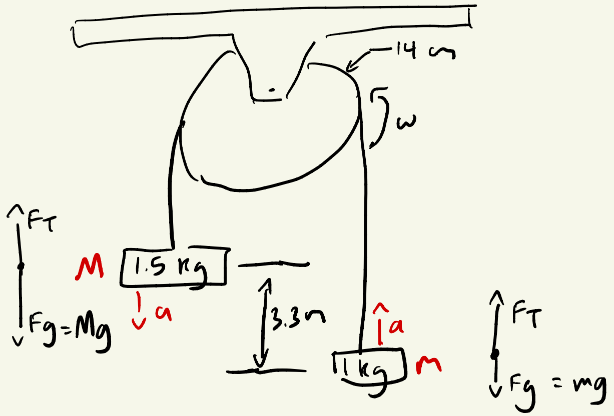
Part C

$$\Sigma F = ma = mg\sin\theta - F_F \qquad F_F = \mu F_N$$

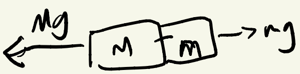
$$ma = mg\sin\theta - \mu F_N \quad \& \quad mg\cos\theta = F_N$$

$$ma = mg\sin\theta - \mu(mg\cos\theta)$$

Atwood machine Problems



Part A



$$\Sigma F_x = ma = Mg - mg$$

$$(M+m)a = Mg - mg \Rightarrow a = \frac{Mg - mg}{M+m}$$

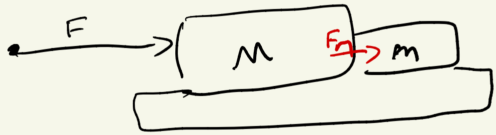
Part B

$$\Sigma F_y = ma = F_T - mg$$

mass of little block

$$F_T = ma + mg$$

Force Between Two Blocks Problem



F pushes M+m

F_m pushes m

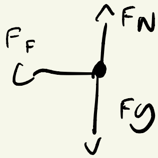
$$\frac{m}{m_{total}} = \frac{m}{M+m} = \frac{1}{10}$$

m is $\frac{1}{10}$ of m_{total}

$$F_m = \frac{1}{10} F$$

Two Cars Braking Problem

$\Sigma F_y = 0 = F_N - F_g \Rightarrow F_N = F_g$



$\Sigma F_x = ma = F_F$

$a = \frac{F_F}{m}$

$F_F = \mu F_N$

$= \mu(mg)$

Part A:

$a = \frac{\cancel{\mu mg}}{m}$ (negative)

$a = -\mu g$

Part B:

$a =$ Part A

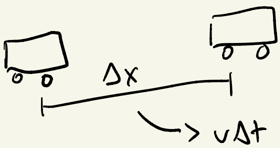
$v_0 =$ given

$v_f = 0$

$\Delta x = ?$

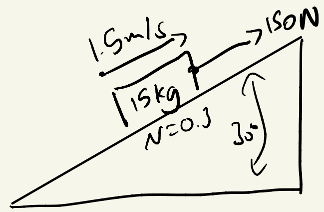
$v_f^2 = v_0^2 + 2a\Delta x$

Part C:

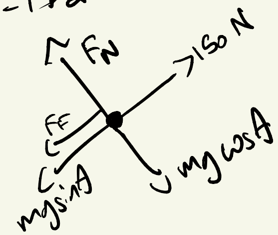


$v = \frac{\Delta x}{\Delta t}$

Crate Pulled Up Ramp Problem



$W = F \cdot d$



Part A

$$\Delta K = W_{Ff} + W_{Fg} + W_{F_{applied}}$$

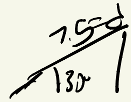
$$W_{Friction} = F_f \cdot d$$

$$= (\mu F_N)\, d = (\mu m g \cos\theta) \cdot d$$

W_{Ff} = negative

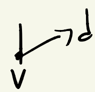
$$W_{Fg} = mg\,\Delta y$$

$$= mg\,d\sin\theta$$



$\Delta y = d\sin\theta$

W_{Fg} = negative



$W_{F_{applied}}$: $W = F \cdot d$ and is positive

Part B

$$\Delta K = K_f - K_0 \qquad K_0 = \tfrac{1}{2} m v_0^2$$

$$K_f = \Delta K + K_0$$

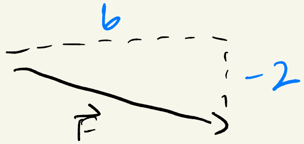
$$K = \tfrac{1}{2} m v^2$$

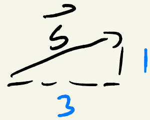
$$\sqrt{\frac{2K}{m}} = v$$

Work done from two vectors problem

$\hat{i} = x \qquad \hat{j} = y$

Part A



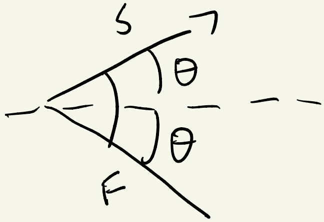


X	Y
$F_x = 6$	$F_y = -2$
$s_x = 3$	$s_y = 1$
$W = 18$	$W = -2$

$$F \cdot s = W$$

$$W_{total} = 18 + (-2) = 16 \text{ J}$$

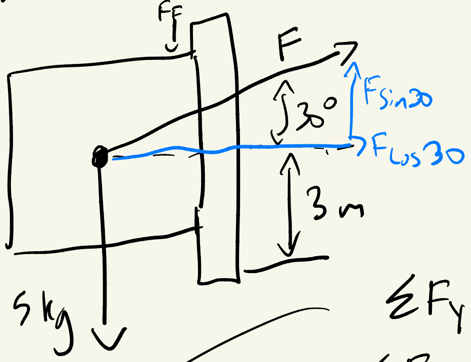
Part B

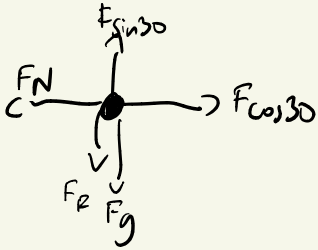


$$\tan^{-1}\left(\frac{O}{A}\right) = \theta$$

for both vectors and add them up

Block on a Wall Problem





$$\Sigma F_y = 0 = F_F + F_g - F\sin 30$$

$$\Sigma F_x = 0 = F_N - F\cos 30$$

$$\Rightarrow F_N = F\cos 30$$

$$\underbrace{F_F}_{\mu F_N} + \underbrace{F_g}_{mg} = F\sin 30$$

$$\mu F\cos 30 + mg = F\sin 30$$

$$\Rightarrow mg = F(\sin 30 - \mu\cos 30)$$

Part A $\Rightarrow F = \dfrac{mg}{\sin 30 - \mu\cos 30}$ and $W = F\cdot d\sin 30$

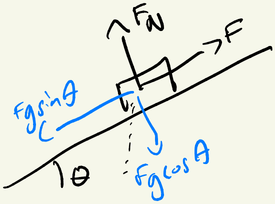
Part B $W = F\cdot d \Rightarrow W_g = F_g\cdot d = mg\cdot d$

Part C $F_N = F\cos 30$

Car Driving Up Pike's Peak Problem

$100\ kW = 100000\ W$

$$P = \frac{\Delta E}{\Delta t}$$



$$\Sigma F_{\parallel} = 0 = F - F_g \sin\theta$$

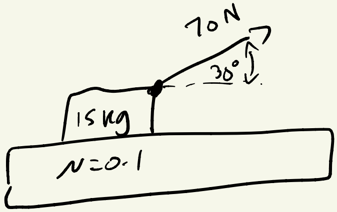
$$\Rightarrow F = F_g \sin\theta$$

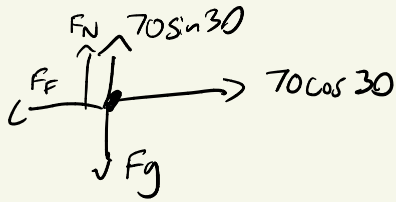
$$F = mg\sin\theta$$

$$P = F \cdot V$$

$$V = \frac{P}{F}$$

Block Dragged on Rough Surface Problem





Part A

$$W = F \cdot d \cos\theta$$

Part B

$$W_{F_F} = F_F \cdot d$$

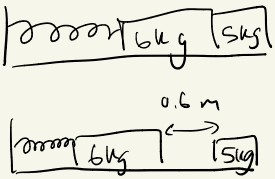
$$F_F = \mu F_N$$

$$\Sigma F_y = 0 = F_N + 70\sin 30 - F_g$$

$$\Rightarrow F_N = F_g - 70\sin 30$$

$$= mg - 70\sin 30$$

Two Blocks and a Spring Problem



Part A $U_E = \frac{1}{2} k \Delta x^2$

Part B speed $\rightarrow K = \frac{1}{2} m v^2$

$W = \Delta K = K_f$ started @ rest

spring friction

$W_{Ff} = F_F \cdot d$

$W = U_E + W_{Ff}$

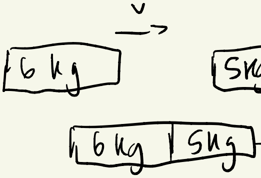
$W_{Ff} = (\mu F_N)(\Delta x)$

$= (+) + (-)$

$W = \Delta K = \frac{1}{2} m v^2$ solve for v

Part C

collision $\rightarrow$ cons of $\vec{p}$



Before(p_0)

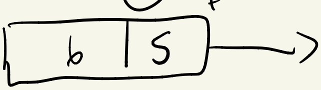
After(p_f)

$$6(v) + 5(0) = (6+5) v_f$$

$$6v = (6+5) \underline{v_f}$$

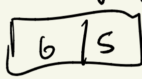
Part D K $K = \frac{1}{2} m v^2$

$v_f = ?$



Q

V=0



$$W_{Ff} = \Delta K$$

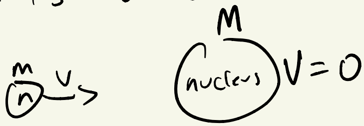
$$F_F \cdot d = \Delta K \Rightarrow d = \frac{K}{F_F} = \frac{K}{\mu (mg)}$$

$F_F = \mu F_N$

$F_N = F_g$

Neutron Collision Problem

Elastic: K is conserved



$$p \rightarrow m_i v_i + \cancel{M_i V_i} = m_f v_f + M_f V_f$$

$$K \rightarrow \frac{m_i v_i^2}{2} + \cancel{\frac{M_i V_i^2}{2}} = \frac{m_f v_f^2}{2} + \frac{M_f V_f^2}{2}$$

Part A: $\quad \text{fraction} = \dfrac{4mM}{(m+M)^2}$

Part B

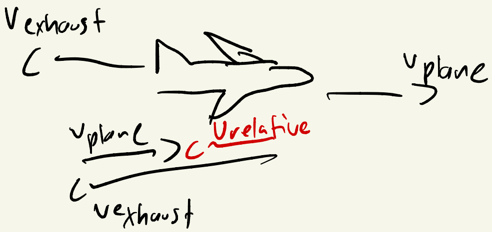
$$(1 - \text{fraction})(K_0) = K$$

Aircraft Thrust Problem

Part A

$$F = \frac{\Delta m}{\Delta t}(v_{exhaust})$$

$$P = F \cdot v = \text{Thrust}(v_{plane})$$



$\frac{\Delta m}{\Delta t}$ fuel

$\frac{\Delta m}{\Delta t}$ air

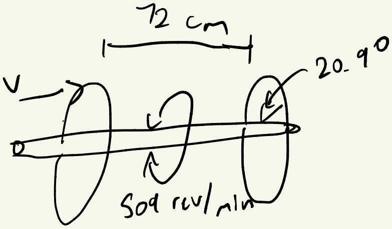
Thrust = fuel part + air part

$$= \frac{\Delta m}{\Delta t}\text{fuel}(v_{ejected}) + \frac{\Delta m}{\Delta t}\text{air}(U_{relative})$$

Part B

$$P = F \cdot V = \text{Thrust}(v_{plane})$$

Rotating Disks Problem



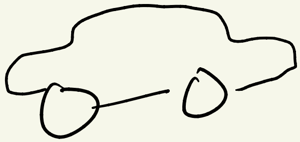
$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\theta}{\omega}$$

$$v = \frac{d}{\Delta t}$$

$$v = \frac{d}{(\theta/\omega)} = \frac{d\omega}{\theta}$$

360° in 1 rev

Car Decelerating Problem



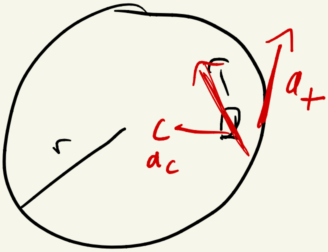
linear → angular

$$\frac{v}{r} = \omega \quad (\text{rad}/\text{s})$$

$$\frac{a}{r} = \alpha \quad (\text{rad}/\text{s}^2)$$

$$\omega_f^2 = \omega_0^2 + 2\alpha \Delta\theta$$

Testing Car Tires Problem

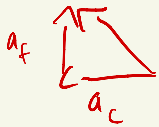


Centripetal acceleration

$$a_c = \frac{v^2}{r}$$

tangential acceleration

given $a_t = \frac{\Delta v}{\Delta t}$



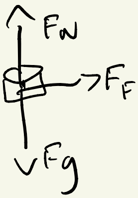
$$a = \sqrt{a_t^2 + a_c^2}$$

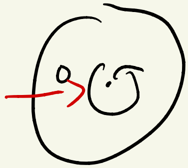
$$\Sigma F = ma = F_F$$

$$ma = \mu F_N$$

$$ma = \mu(mg) \Rightarrow a = \mu g$$

Coin on Turntable Problem



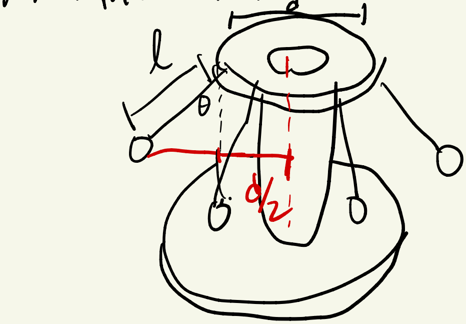


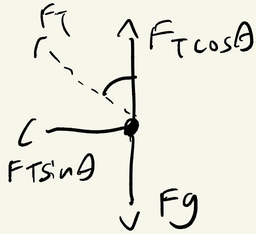
$$\Sigma F_x = \frac{mv^2}{r} = F_F$$

$$\frac{mv^2}{r} = \mu F_N$$

$$\frac{mv^2}{r} = \mu m g$$

Amusement Park Ride Problem





1) Find F_T

$$\sum F_y = 0 = F_T \cos\theta - F_g$$

$$\Rightarrow F_T \cos\theta = mg$$

2) $\sum F_x = \frac{mv^2}{r} = F_T \sin\theta$

$$r = \left(\frac{d}{2}\right) + l \sin\theta$$

$v = ?$

Part A

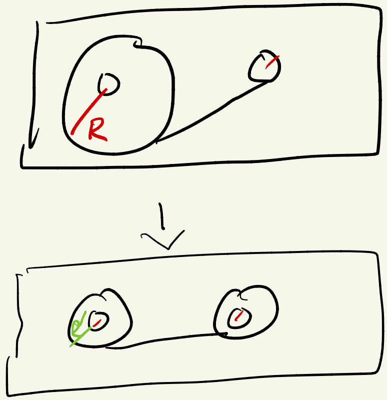
Part B

$$F_T \cos\theta = F_g$$

$$F_T \cos\theta = mg$$

$(m_{seat} + m_{child})$

VCR Tape Problem



R: radius of full reel
r: inner radius of reel
R': radius of reels when $\omega_1 = \omega_2$

$$\pi R'^2 - \pi r^2 = \frac{1}{2}\left(\pi R^2 - \pi r^2\right)$$

$$\Rightarrow R'^2 - r^2 = \frac{R^2}{2} - \frac{r^2}{2}$$

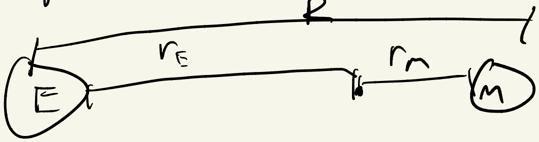
$$\Rightarrow R'^2 = \frac{R^2}{2} + \frac{r^2}{2} = \frac{R^2 + r^2}{2}$$

$$\Rightarrow R' = \sqrt{\frac{R^2 + r^2}{2}}$$

$$v = r\omega \Rightarrow \omega = \frac{v}{r} \quad \& \quad v = \frac{l}{\Delta t}$$

hr to sec

Apollo Spacecraft Problem



$R = r_E + r_m$

Part A

$$F_E = F_M \Rightarrow \frac{GM_E m}{r_E^2} = \frac{GM_m m}{r_m^2}$$

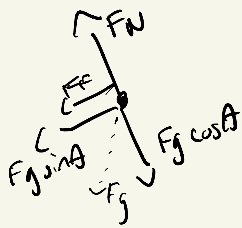
$$\Rightarrow M_E r_m^2 = M_m r_E^2 \Rightarrow \sqrt{\frac{M_m}{M_e}} = \frac{r_m}{r_E}$$

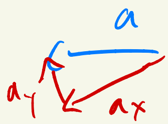
$$R = r_E + r_E\sqrt{\frac{M_m}{M_E}}$$

$$= r_E\left(1 + \sqrt{\frac{M_m}{M_E}}\right) \Rightarrow r_E = \frac{R}{1+\sqrt{\frac{M_m}{M_E}}}$$

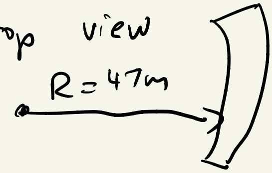
Part B: Trivial

Car on a Curve Problem

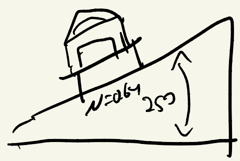




top view



rear view



$$a_x = \frac{v^2}{r}\cos\theta$$

$$a_y = \frac{v^2}{r}\sin\theta$$

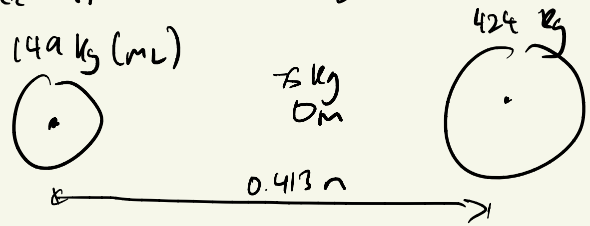
$$\sum F_x = \frac{v^2}{r}\cos\theta = F_F + F_g\sin\theta$$

$$\sum F_y = \frac{v^2}{r}\sin\theta = F_N - F_g\cos\theta$$

Solve for v

$$\boxed{v = \frac{(rg)(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$

Force from Two Large Masses Problem



$r = \frac{0.413}{2}$

$$F_B = \frac{G M_B m}{r^2} \qquad F_L = \frac{G M_L m}{r^2}$$

$$F = \frac{G M_B m}{r^2} - \frac{G M_L m}{r^2} = \frac{G m}{r^2}\left(M_B - M_L\right)$$ (Part A)

Part B

$F_B = F_L$

$d = 0.413$ m

assume r is distance from big one

so $d - r$ is distance from smaller one

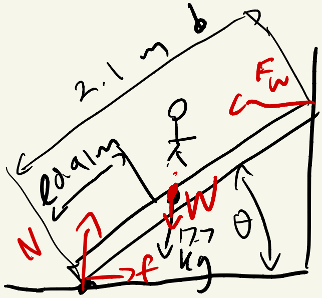
$$\frac{G M_B \cancel{m}}{r^2} = \frac{G M_L \cancel{m}}{(d-r)^2} \Rightarrow \frac{M_B}{r^2} = \frac{M_L}{(d-r)^2} \Rightarrow M_B (d-r)^2 = M_L r^2$$

$$\Rightarrow (d-r)\sqrt{M_B} = r\sqrt{M_L} \Rightarrow d\sqrt{M_B} - r\sqrt{M_B} = r\sqrt{M_L}$$

$$d\sqrt{M_B} = r\sqrt{M_L} + r\sqrt{M_B}$$

$$\Rightarrow r = \frac{d\sqrt{M_B}}{\sqrt{M_L} + \sqrt{M_B}}$$

Man on a ladder problem



$\mu = 0$

$\Sigma F_x = 0 = F_w - f \Rightarrow F_w = f$

$\Sigma F_y = 0 = N - w \Rightarrow N = w$

$\Sigma \tau = 0 = F_w d \sin\theta - w \ell \cos\theta$

$F_F = \mu F_g$
(mg)

$F_w d \sin\theta = w \ell \cos\theta$

$(\mu mg) d \sin\theta = mg \ell \cos\theta$

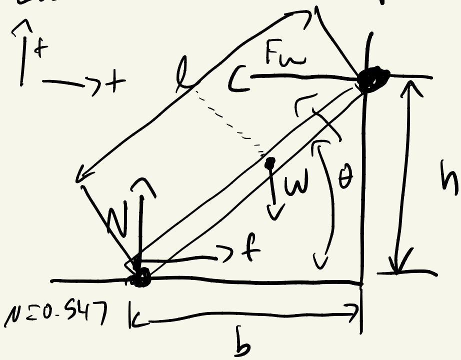
$\mu d \sin\theta = \ell \cos\theta$

$\mu d \tan\theta = \ell$

$\tan\theta = \frac{\ell}{\mu d}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\ell}{\mu d}\right)$

Ladder on a wall problem



$$\Sigma F_x = 0 = f - F_w$$

$$\Sigma F_y = 0 = N - W$$

$$\Sigma \tau = 0 = F_w \ell \sin\theta - w\left(\frac{\ell}{2}\right)\cos\theta$$

x: $F_w = f = \mu F_N$

τ: $F_w \ell \sin\theta = w\left(\frac{\ell}{2}\right)\cos\theta$

$$F_w \sin\theta = \frac{w}{2}\cos\theta$$

y: $N = w$

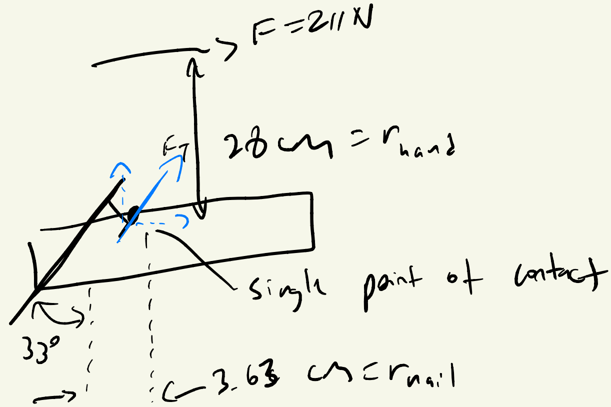
$$F_w \sin\theta = \frac{w}{2}\cos\theta$$

$$(\mu N)\sin\theta = \frac{w}{2}\cos\theta$$

$$\mu N \sin\theta = \frac{N}{2}\cos\theta$$

$$\mu \sin\theta = \frac{\cos\theta}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Hammer and Nail Problem



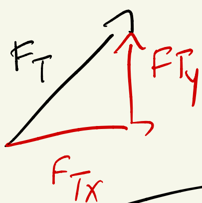
$\tau_{hand} = \tau_{claw}$

$$F(0.28) = F_{nail}(0.0363)\sin\theta \qquad \text{(Part A)}$$

Part B:

$$\Sigma F_x = 0 = F_H - F_N \sin\theta + F_{T_x}$$

$$\Sigma F_y = 0 = F_{T_y} - F_N \cos\theta$$

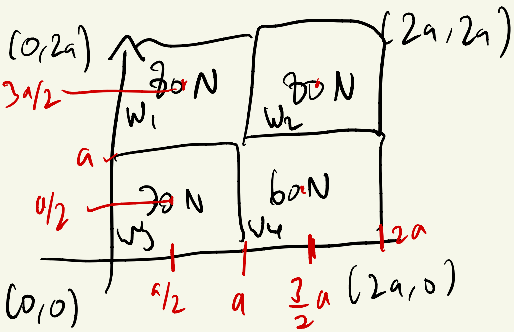


$$F_{T_x} + F_N \sin\theta = F_H$$

$$F_N \cos\theta = F_{T_y}$$

$$F_T = \sqrt{F_{T_x}^2 + F_{T_y}^2}$$

Center of Mass 4 Plates Problem



Part A

$$X_{cg} = \frac{\Sigma_i W_i x_i}{\Sigma_i W_i}$$

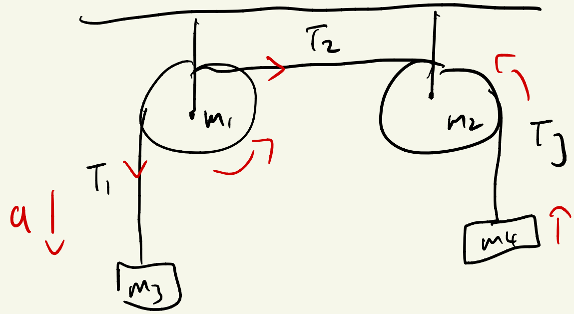
$$X_{cg} = \frac{W_1\left(\frac{a}{2}\right) + W_3\left(\frac{a}{2}\right) + W_2\left(\frac{3a}{2}\right) + W_4\left(\frac{3a}{2}\right)}{W_1 + W_2 + W_3 + W_4}$$

Part B

$$Y_{cg} = \frac{\Sigma_i W_i Y_i}{\Sigma_i W_i}$$

$$Y_{cg} = \frac{W_1\left(\frac{3a}{2}\right) + W_2\left(\frac{3a}{2}\right) + W_3\left(\frac{a}{2}\right) + W_4\left(\frac{a}{2}\right)}{W_1 + W_2 + W_3 + W_4}$$

Blocks and Pulley Problem



For each pulley, $I = mr^2$

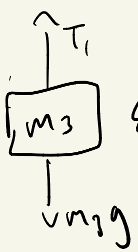
$$\Sigma \tau = I\alpha = (mr^2)\left(\frac{a}{r}\right) = mra$$

Left pulley:

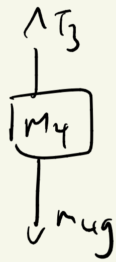
$$\Sigma \tau = m_1 r_1 a = T_1 r_1 - T_2 r_1 \Rightarrow m_1 a = T_1 - T_2$$

Right pulley:

$$\Sigma \tau = m_2 r_2 a = T_2 r_2 - T_3 r_2 \Rightarrow m_2 a = T_2 - T_3$$



$$\Sigma F = m_3 a = m_3 g - T_1$$



$$\Sigma F = m_4 a = T_3 - m_4 g$$

$$m_1 a + m_2 a + m_3 a + m_4 a = m_3 g - m_4 g$$

Energy of Merry-go round problem

1) Find I $\quad I_{cylinder} = \frac{1}{2}mr^2$

$346\ N \Rightarrow \frac{346}{9.8} = m$

2) Find α from τ:

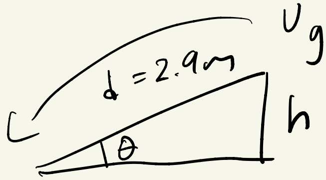
$a = \frac{\Sigma F}{m} \rightarrow \alpha = \frac{\Sigma \tau}{I}$ $\quad \tau = F \cdot r$

3) Use α to solve for ω_f

$\omega_f = \omega_0 + \alpha \Delta t$

4) $K = \frac{1}{2}mv^2 \Rightarrow K_{rot} = \frac{1}{2} I \omega^2$

Rolling Basketball Problem



$h = d\sin\theta$

$I_{sphere} = \frac{2}{3}mr^2$

$v = \omega r$

$U_g = K_{translational} + K_{rotational}$

$mg\Delta y = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v}{r}\right)^2$

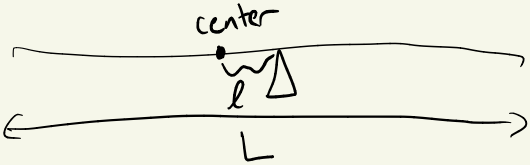
$\Rightarrow gh = \frac{5}{6}v^2 = gd\sin\theta$

$\Rightarrow v = \sqrt{\frac{6gd\sin\theta}{5}}$

$\frac{v_f}{2}\Delta t = \Delta x$

Dishonest Pan Balance Problem

W = actual weight
W' = dishonest weight



$$\% = \frac{\text{difference}}{\text{"dishonest"}}$$

$$W\left(\frac{L}{2} - \ell\right) = W'\left(\frac{L}{2} + \ell\right)$$

$$\frac{W}{W'} = \frac{\frac{L}{2} + \ell}{\frac{L}{2} - \ell} = \frac{L + 2\ell}{L - 2\ell}$$

$$\frac{W - W'}{W'} = \frac{L + 2\ell}{L - 2\ell} - 1$$

Pitched Baseball Problem

$$I_{sphere} = \frac{2}{5}mr^2$$

$$\frac{K_{rot}}{K_{trans}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)mr^2\omega^2}{\frac{1}{2}mv^2}$$

$$= \frac{2r^2\omega^2}{5v^2}$$

Rotating Stool Problem

$$L_0 = L_f$$
$$I_0 w_0 = I_f w_f$$

Initial I: $\quad Stool + (2mr^2)$

Final I: $\quad Stool + 2(mr^2)$

Merry-go-round Consv. Ang. Momentum

$L_0 = L_f$ $I_{cyl} = \frac{1}{2}mr^2$

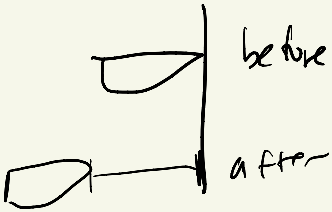
$I_0 \omega_0 = I_f \omega_f$

$I_0 = I_{cyl} + I_{man} = I_{cyl} + mr^2$

$I_f = I_{cyl}$

1 rev has 2π rad

Man Walking in Boat



$$\Delta x_{cm} = \frac{m_m x_m + m_b x_b}{m_m + m_b}$$

$$= \frac{m_m \Delta x_m + m_b \Delta x_b}{m_m + m_b} = 0$$

$$m_m \Delta x_m + m_b x_b = 0$$

$$m_b = m_m \left(\frac{\Delta x_m}{-\Delta x_b} \right)$$

$\Delta x_m = ($ l of boat $) - ($ d to pier $)$

$\Delta x_b = ($ d to pier $)$

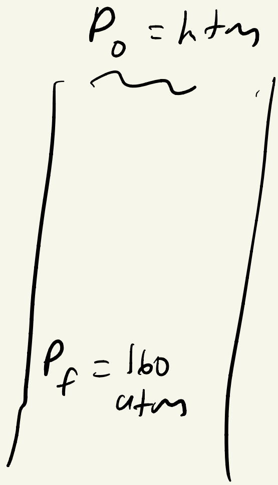
$$\Delta x_m = l - \Delta x_b$$

Bulk Modulus Problem

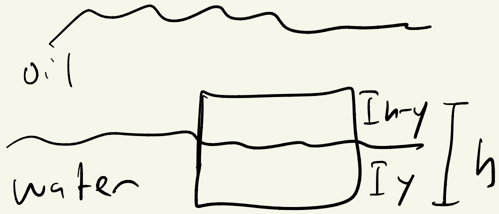
$\rho_0 = 1025$

$\Delta P = 160 - 1 = 159$ atm

$$\Delta \rho = \frac{\rho_0 \Delta P}{B}$$



Block in Oil and water



$$F_{B\,oil} + F_{B\,water} = F_g \qquad F_B = \rho V g$$

$$\rho_{oil} V g + \rho_{water} V g = mg \qquad \rho = \frac{m}{V} \qquad m = \rho V$$

$$V = Ah$$

$$\rho_{oil} V + \rho_{water} V = \rho_{block} V$$

$$\rho_{oil}(A)(h-y) + \rho_{water}(y)(A) = \rho_{block}(A)(h)$$

$$\rho_{oil}(h-y) + \rho_{water}(y) = \rho_{block}(h)$$

$$\Rightarrow \rho_{water}(y) - \rho_{oil}(y) = \rho_{block}(h) - \rho_{oil}(h)$$

$$\Rightarrow y(\rho_{water} - \rho_{oil}) = \rho_{block}(h) - \rho_{oil}(h)$$

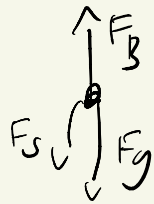
Block on Spring underwater

$F_B = \rho V g$

$F_s = k \Delta x$

$\uparrow$

$\Delta L = \Delta x$



$\Sigma F = 0 = F_g + F_s - F_B$

$F_s = F_B - F_g$

$k \Delta L = \rho V g - mg$

$\rho_b = \frac{m_b}{V_b}$

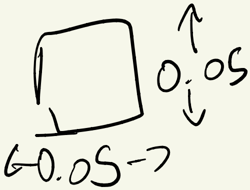
$V_b = \frac{m_b}{\rho_b}$

$$k \Delta L = \rho_{water}\left(\frac{m_{block}}{\rho_{block}}\right) g - m_{block}(g)$$

$\rho_{water} = 1000 \ kg/m^3$

Basic Pressure Problems

$P = \frac{F}{A}$ $F = mg$



$A = (0.05^2) \times 2$

then Convert Pa to psi

Nuclear Ore

$P = \frac{F}{A}$ $P = 0.2$ atm → Pa

1 atm = 101300 Pa

Pressure Under the Ocean

$$P = P_{atm} + \rho g h$$

$$\rho_{water} = 1000 \ kg/m^3$$

$$P = 3 \times P_{atm}$$

Object Immersed in Water and Oil

$F_B = \Delta$ in weight air $\rightarrow$ water

$F_B = \rho V g$

$\rho_{water} = 1000 \text{ kg/m}^3$

$g = 9.8 \text{ m/s}^2$

$\rho_{obj} = \frac{m_{obj}}{V_{obj}}$

$F_g = mg$

$\frac{F_g}{g} = m$

Part B

$F_B = \rho_{oil} V g$

300 N out

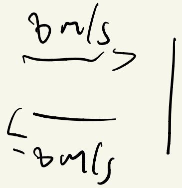
275 N in

Hailstones On Windshield

Part B $= \frac{F}{A} = P$

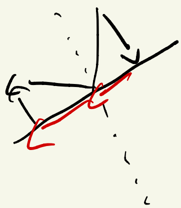
elastic collision → $V_f = V_o$

$\Delta V = 2v$



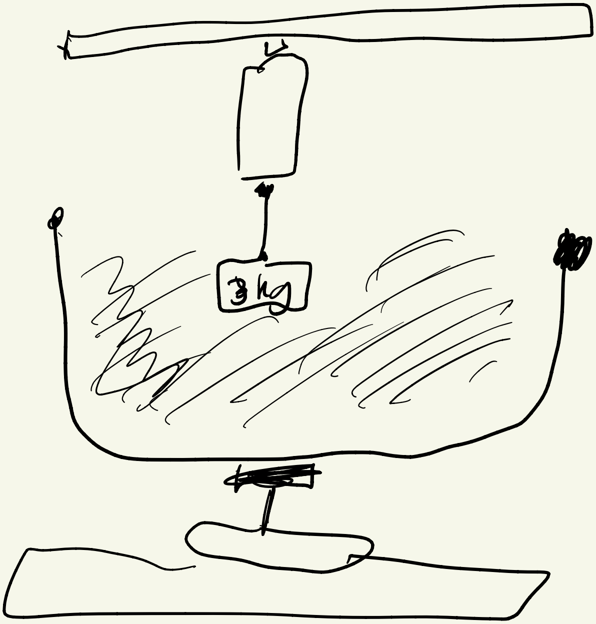
$F\Delta t = m\Delta V$

$$F = \frac{m\Delta v}{\Delta t} = \frac{m(16)\sin\theta(\#\text{ of stones})}{(\Delta t)}$$



$$\frac{500 \text{ stones}}{30 \text{ s}} = \# \frac{\text{stones}}{\text{s}}$$

Two Scales Buoyancy Problem



$F_B = \rho V g$

$\rho = 1000$

$g = 9.8$

$\rho = \frac{m}{V}$

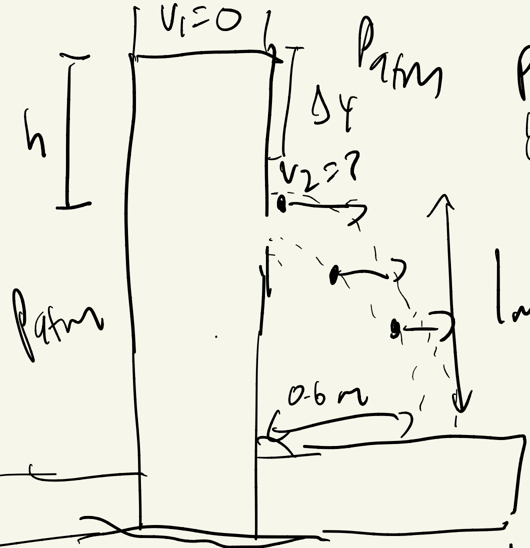
$F_S = F_g - F_B$

$3(9.8)$

total weight

$= m_w g + m_b g + m_a g - F_S$

Water Squirting from Tank



$$P_1 + \rho g \Delta y + \frac{1}{2}\rho v_1^2 = P_2 + \rho g \Delta y + \frac{1}{2}\rho v_2^2$$

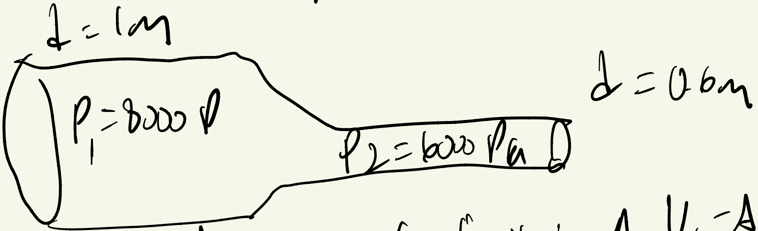
0 0 0

$$\rho g \Delta y = \frac{1}{2}\rho v_2^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(1)}{9.8}} = \Delta t$$

$$v_x = \frac{\Delta x}{\Delta t}$$

Oil in Horizontal Pipe



Flow Rate = Av

Continuity: $A_1 v_1 = A_2 v_2$

$\downarrow$ $v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{r_2^2} = \frac{r_1^2}{r_2^2} v_1$

Bernoulli:

$$P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho v_2^2 = P_2 + \frac{1}{2}\rho\left(\frac{r_1^2}{r_2^2} v_1\right)^2$$

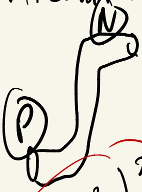
$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(\frac{r_1^4}{r_2^4} v_1^2\right) - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{r_1^4}{r_2^4} - 1\right)$$

Plug into continuity

Area = $\pi\left(\frac{d}{2}\right)^2$

Firenga Hole Problem



$A = \left(\frac{2}{2}\right)^2 \pi$

$A = \left(\frac{3}{2}\right)^2 \pi$

r_p

$$P_N + \rho g \Delta y + \frac{1}{2} \rho v_N^2 = P_p + \cancel{\rho g \Delta y} + \frac{1}{2} \rho v_p^2$$

$$\frac{1}{2} \rho v_N^2 - \frac{1}{2} \rho v_p^2 = P_p + P_N - \rho g \Delta y$$

$$\frac{\rho}{2}\left(v_N^2 - v_p^2\right) = P_p - P_N - \rho g \Delta y$$

$$\boxed{v_N^2 - v_p^2} = \frac{2(P_p - P_N - \rho g \Delta y)}{\rho}$$

297.854 kPa = $P_p - P_N$

$$A_p v_p = A_N v_N \Rightarrow (\pi r_p^2) v_p = (\pi r_N^2) v_N$$

$$\frac{v_p}{v_N} = \frac{\pi r_N^2}{\pi r_p^2} \Rightarrow v_p = v_n \frac{r_N^2}{r_p^2}$$

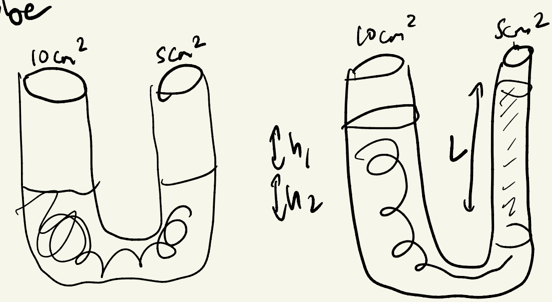
Let $v_N^2 - v_p^2 = x$

$$x = v_N^2 - v_p^2 = v_N^2 \left(v_n \frac{r_N^2}{r_p^2}\right)^2$$

$$\Rightarrow x = v_N^2 - v_N^2 \frac{r_N^4}{r_p^4} \Rightarrow x = v_N^2\left(1 - \frac{r_N^4}{r_p^4}\right)$$

$$v_N = \sqrt{\text{something}}$$

U Tube



Find L

V of cylinder = $A \cdot L$ $\qquad$ $\rho = \frac{m}{V}$

(m → cm)

2) $P_H = P_L$

$\rho g(h_1 + h_2) = \rho g L$

$\rho g h_1 \left(1 + \frac{A_H}{A_L}\right) = \rho g L$

$A_H \cdot h_1 = A_L \cdot h_2$

$\Rightarrow h_2 = \frac{A_H h_1}{A_L}$

Temp of Air

$$v = v_0 \sqrt{\frac{T}{273}}$$

$v = f\lambda$

$$f\lambda = v_0 \sqrt{\frac{T}{273}}$$

$$\frac{f\lambda}{v_0} = \sqrt{\frac{T}{273}} \implies T = \underline{\hspace{3em}}$$

$K - 273 = {}^\circ C$

Rock Band

$A = \log_{10} B$

$B = 10^A$

$\beta = 10 \log\left(\frac{I}{I_0}\right)$

$\underbrace{\frac{\beta}{10}}_{A} = \underbrace{\log\left(\frac{I}{I_0}\right)}_{B}$

$\left(\frac{I}{I_0}\right) = 10^{\beta/10}$

$\frac{I}{I_0} = \frac{r_0^2}{r^2}$

$r\sqrt{\left(\frac{I}{I_0}\right)} = r_0 = r\sqrt{10^{\beta/10}}$

$I_0 = 10^{-12}\ W/m^2$

$\beta = 80$

$r = 5m$

$I \propto \frac{1}{r^2}$

$Ir^2 = I_0 r_0^2$

$\frac{I}{I_0} = \frac{r_0^2}{r^2}$

Intensity of Chorus

Total Intensity = Sum of individual intensities

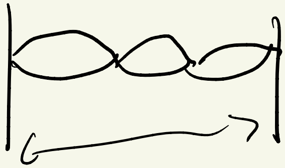
$$I_n = n I_1$$

$$\beta_n = 10 \log \frac{n I_1}{I_0}$$

$$= 10 \log n + \underbrace{10 \log \left(\frac{I_1}{I_0} \right)}_{\beta_n}$$

String Harmonics

2 harmonics = 1λ



70 cm

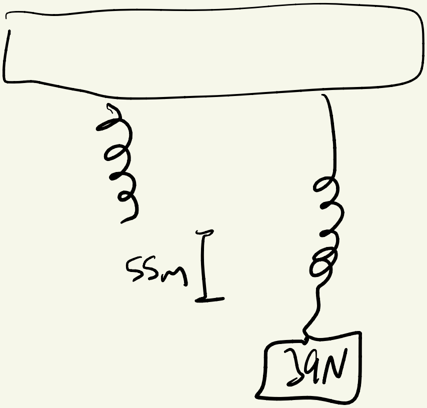
(a) $v = f\lambda$ $\lambda = \frac{2}{3}l = \frac{2}{3}(70)$

$F = \frac{mv^2}{l}$

(b) $f_n = n f_f$

$v = \sqrt{\frac{F}{\mu}}$ $\mu = \frac{m}{l}$

Spring Constant Problem

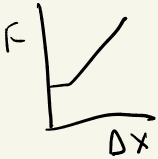


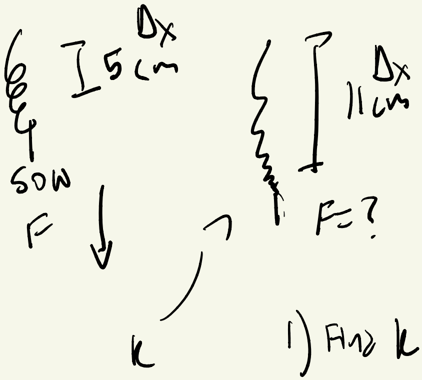
$F = k\Delta x$

k = spring constant

String Stretched Twice Problem

$$F = k\Delta x$$

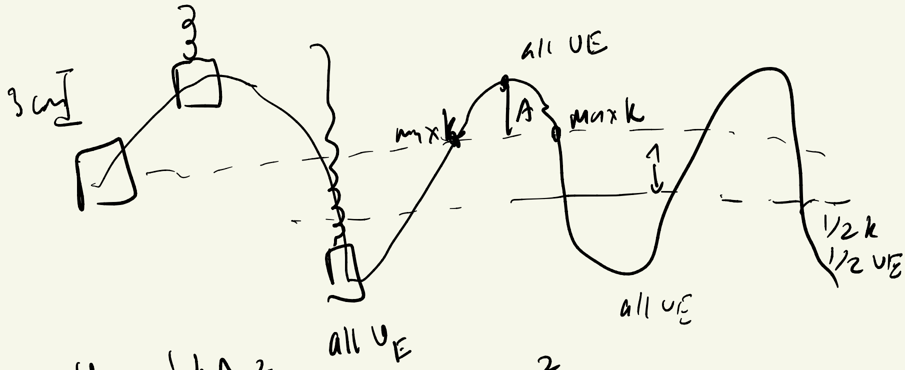




1) Find k

2) Use k to get F

Half of Max Speed SHM



$U_E = \frac{1}{2}k\Delta x^2$

$k = m\omega^2$

$U_E = \frac{1}{2}(m\omega^2)\Delta x^2$

$\frac{1}{2}m\omega^2\Delta x^2 = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2\Delta x^2$

$V = -A\omega\sin(\omega t)$

$V = \left(\frac{A\omega}{2}\right)$

$\omega^2\Delta x^2 = v^2 + \omega^2\Delta x^2$

$\omega^2 A^2 = v^2 + \omega^2\Delta x^2$

$\omega^2 A^2 = \left(\frac{A\omega}{2}\right)^2 + \omega^2\Delta x^2$

$A^2 = \frac{A^2}{4} + \Delta x^2$

$\Delta x = ?$

Tension in phone cord

$4\,m = l$

$\Delta x = 4(2l)$

$$v = \frac{\Delta x}{\Delta t} = \frac{4(2l)}{\Delta t}$$

$$v = \sqrt{\frac{F}{\mu}} \qquad \mu = m/l$$

$$v^2 = \frac{F}{\mu} \Rightarrow v_2 = \frac{F}{m/l} = \frac{Fl}{m}$$

$$v^2 = \frac{Fl}{m}$$

Steel Track Expansion

(a) $\Delta L = \alpha L_0 \Delta T$

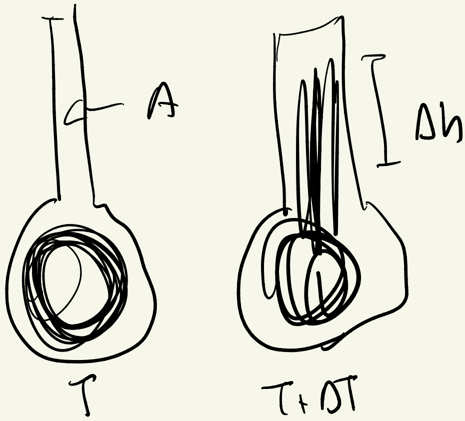
$T_f - T_0$

$40 - 0$

(b) $Y = \dfrac{\text{stress}}{\text{strain}}$ $\text{Stress} = Y\left(\dfrac{\Delta L}{L_0}\right)$

$\text{strain} = \dfrac{\Delta L}{L_0}$

Mercury Thermometer



$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{cylinder} = \pi r^2 h$

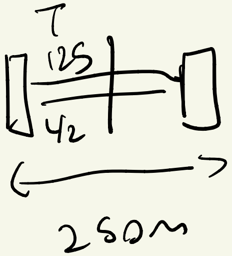
$\Delta V = \alpha V_0 \Delta T$

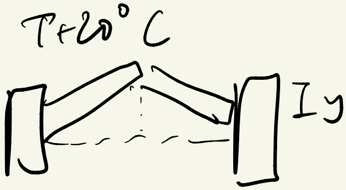
$\Delta V = \alpha \left[\frac{4}{3} \pi \left(\frac{D}{2} \right)^3 \right] (\Delta T)$

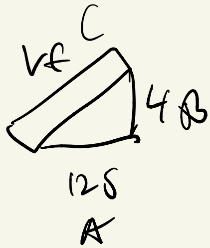
$\Delta V = V_{cylinder}$

$\Delta V = \pi r^2 h$

Concrete Expansion







$$A^2 + B^2 = C^2$$

$$\Delta L = \alpha L_0 \Delta T$$

$$L_f = L_0 + \Delta T$$

Molecule Escaping Earth

(a) $K = U_g$

$\frac{1}{2}mv^2 = -\frac{GmM}{r}$

$$F_g = \boxed{\frac{GMm}{r^2}} \rightarrow g$$

$$g = \frac{GM}{R}$$

$$K = \frac{GmM}{R} \qquad g = \frac{GM}{R}$$

(b) $m_{O_2} = \frac{2(16)}{6.02 \times 10^{23}}(1000) = m \text{ of } O_2$

$N = 10$

$k_B = 1.38 \times 10^{-23}$

$K = \frac{3}{2} N k_B T$

Railroad Spike

$$K = U$$

$$0.85\left(\frac{1}{2}mv^2\right) = U$$

Translational KE of Oxygen

$$P = 1 \text{ atm} = 101,500(1) = 101,500 \text{ P}$$

$$T = 0°C = 273 \text{ K}$$

$$V = 1 L = 0.001 m^3 \qquad R = 8.31$$

$$1000 L = 1 m^3$$

1) Find n

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

2) $U = \frac{3}{2} nRT$

Pressure in a tire

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$P_1 = 20\ psi + 14.7 = 34.7\ psi$

$T_1 = -5^\circ C + 273 = 268\ K$

$T_2 = 20^\circ C + 273 = 293\ K$

$P_2 - 14.7 = P_{2\,gauge}$

Avg KE of a Gas Molecule

$P = 8\ atm(101300)$

$V = 5/1000$

$PV = nRT \longrightarrow K = \frac{3}{2} N k_B T$

1

Climbing to work off Cake

$$Cal \cdot 4186 \ J/cal = Joules$$

Food Energy $= U_g = mg\Delta y$

Bullet Fired into Steel

$$K = Q$$

$$K = \frac{1}{2}mv^2$$

$$Q = mc\Delta T$$

$$\frac{1}{2}mv^2 = mc\Delta T$$

$$\frac{\frac{1}{2}v^2}{c} = \Delta T$$

Glass Thermometer in Hot Water

1 = glass

$m_1 = 0.3$ kg

$c_1 = 0.2$

$T_{0_1} = 25$

2 = water

$m_2 = 0.200$ kg

$c_2 = 1$

$T_{0_2} = 95$

$$m_1 c_1 \Delta T_1 = m_2 c_2 \Delta T_2$$

$$m_1 c_1 (T_f - T_{0_1}) = m_2 c_2 (T_f - T_{0_2})$$

$$T_f = \frac{m_2 c_2 T_{0_2} + m_1 c_1 T_{0_1}}{m_1 c_1 + m_2 c_2}$$

Water Freezing ~~on~~ to Ice Cube

$$Q_{\text{freeze } H_2O} = \text{raise T of } H_2O \text{ from } -90° \text{ to } 0°$$

$$\Delta Q = \underset{(\text{ice})}{m_i C \Delta T} = \underset{(\text{water})}{m_w L_f}$$

$$m_w = \frac{m_i C \Delta T}{L_f}$$

Hot ingot in water

metal (1)	water (2)
$m_1 = 0.05$	$m_2 = 0.4$
$c_1 = ?$	$c_2 = 4186$
$T_{01} = 200$	$T_{02} = 20$

$$T_f = 22.4$$

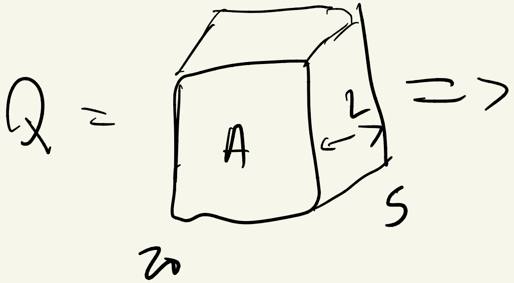
$$Q_{lost} = Q_{gained}$$

metal = water

$$-m_1 c_1 \Delta T_1 = m_2 c_2 \Delta T_2$$

$$c_1 = \frac{m_2 c_2 (T_f - T_{02})}{m_1 (T_f - T_{01})}$$

Brick wall Conductivity



$$Q = kA \frac{\Delta T}{L} \Delta t$$

h → s

h · 60 · 60

h · 3600

$m = 1 \times 10^6$

Ice Added to Tea

$1\ L = 0.001\ m^3 = 1000\ g$

$M_w = 1000\ g$

$C_w = 1$

$\Delta T_w = 20$

$\Delta T_i = 10$

$L_i = 79.7$

$Q_{\text{lost by water}}$

$= Q_{\text{melt ice}} + Q_{\text{raise } T}$

$m_w C_w \Delta T_w = m_i Lf_i + m_i C_w \Delta T_{ice}$

$m_w C_w \Delta T_w = m_i \left(Lf_i + C_w \Delta T_i \right)$

Conductivity of Insulator

cm → m

$$\frac{Q}{\Delta t} = kA\left(\frac{\Delta T}{L}\right)$$

$$P = \frac{Q}{\Delta t}$$

$$P = kA\left(\frac{\Delta T}{L}\right)$$

Tea in the Sun

$$Q_{\text{lost by water}} = Q_{\text{melt ice}} + Q_{\text{ice } \Delta T}$$

$$m_w c_w \Delta T_w = m_i L_{fi} + m_i c_w \Delta T_i$$

$$\Rightarrow m_i = \frac{m_w c_w \Delta T_w}{(L_{fi} + c_w \Delta T_i)}$$

$$m_{\text{total ice}} - m_i = m_{i\,\text{left}}$$

$m_w =$

$m_{i\,\text{total}} =$

$c_w = 4186$

$L_{fi} = 3.33 \times 10^5$

$\Delta T_w = 0.3$

$\Delta T_i = 31.7$

Three liquids

mix 1 and 2:

$\Delta T_1 = 7$

$\Delta T_2 = 3$

$$m_1 c_1 \Delta T_1 = m_2 c_2 \Delta T_2$$

$$\Rightarrow c_2 = \frac{7}{3} c_1$$

mix 2 and 3

$\Delta T_2 = 8$

$\Delta T_3 = 2$

$$m_2 c_2 \Delta T_2 = m_3 c_3 \Delta T_3$$

$$\Rightarrow c_3 = 4 c_2$$

$$\Rightarrow c_3 = \frac{28}{3} c_1$$

mix 1 and 3

$$m_1 c_1 \Delta T_1 = m_3 c_3 \Delta T_3$$

$$m_1 c_1 (T_f - T_{01}) = m_3 c_3 (T_{03} - T_f)$$

$$m_1 c_1 T_f + m_3 c_3 T_f = m_3 c_3 T_{03} + m_1 c_1 T_{0\cancel{f}1}$$

$$T_{\cancel{f}} = \frac{\left(\frac{28}{3}\right) T_{03} + T_{01}}{1 + \left(\frac{28}{3}\right)}$$

Ice in a Copper Cup

$m_i = 0.040$

$m_w = 0.560$

$m_{cu} = 0.08$

$c_i = 2090$

$c_w = 4186$

$c_{cu} = 387$

$L_f = 3.33 \times 10^5$

1) Heat Gained by Ice

$$H_1 = m_i c_i (78) + m_i L_f + m_i c_w (T_f)$$

$$\underline{H_1} + m_i c_w T_f$$

2) Heat Lost by Water

$$H_2 = m_w c_w (25 - T_f) = m_w c_w 25 - m_w c_w T_f$$

$$\underline{H_2} - m_w c_w T_f$$

$$H_1 = H_2 + H_3$$

3) Heat Lost by Copper

$$H_3 = m_{cu} c_{cu} (25 - T_f) = m_{cu} c_{cu} 25 - m_{cu} c_{cu} T_f$$

$$H_3 - m_{cu} c_{cu} T_f$$

$$T_f = \frac{H_2 + H_3 - H_1}{m_i c_w + m_w c_w + m_{cu} c_{cu}}$$

Refrigerator Power

$$P = \frac{W}{\Delta t} \qquad \frac{\Delta Q}{\Delta t} = \frac{kA(T_H - T_C)}{L}$$

$$W = \frac{Q_C(T_H - T_C)}{C\,T_C}$$

0.075

$$P = \frac{W}{\Delta t} = \frac{Q_C(T_H - T_C)}{\Delta t\,(C\,T_C)}$$

$$= \frac{kA(T_H - T_C)}{L}\,\frac{(T_H - T_C)}{C(T_C)}$$

$$= \frac{kA(T_H - T_C)^2}{(L)(C)(T_C)}$$